Written Exam Economics Winter 2017-2018

Financial Markets

Date (from 17 February 2018 to 19 February 2018)

This exam question consists of 6 pages in total

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Focus on Exam Cheating

In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

● 1. A warning

● 2. Expulsion from the examination

● 3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Problem 1

(a). Explain why it might be valuable for market makers to observe the order flow, and why this might be bad for traders.

(b). In limit order books, traders choose themselves whether to provide liquidity (post limit orders) or to take liquidity (post market orders). Thus, in some sense, traders choose between being market makers and speculators, in the language of the Glosten/Milgrom model. What might affect their choice of becoming one or the other?

(c). Explain why it is that, given a set of limit orders, a uniform price auction will always be efficient in the sense that after the auction takes place, the sellers who did not sell must necessarily have posted a higher price than those sellers who did sell, and the buyers who did not buy must necessarily have posted a lower price than those buyers who did buy. Compare this with other market mechanisms (other auction types, continuous limit-order books, etc.). What may be an argument for not always prioritizing efficiency?

Problem 2

In this question we use a Glosten-Milgrom style model to analyze information acquisition by market makers. The model has the following features:

- **Setup.** Consider the market for a risky asset with value V. For simplicity, $V = 1$ with probability $\frac{1}{2}$ and $V = 0$ with probability $\frac{1}{2}$.
- **Market makers.** There are two market makers (MMs, indexed by $n = 1, 2$). The MMs simultaneously set ask and bid prices a_n and b_n. If MMn gets to trade, his profit is $a_n - V$ if the incoming order is a buy order, and $V - b_n$ if the incoming order is a sell order. If he does not trade, his profit is zero.
- **Price priority.** There is price priority on the market, such that the MM with the best price always receives the incoming order. If the MMs set the same price, each gets the order with probability 1/2. Let the 'best prices' be denoted by $\hat{a} := \min_n a_n$ and $\hat{b} := \max_n b_n$.
- **Traders.** There is a single trader, who is *always a noise trader* and buys/sells one unit with probability 1/2.
- **Information acquisition.** Before trading takes place, each MM can learn V at a cost of $c > 0$. Neither the decision of whether to learn V nor the value of V is observed by the other MM. Let p_n be the probability that MMn acquires information.

We will look for a perfect Bayesian equilibrium (PBE). We now proceed to solve the model. Since there is only noise trading in the model and since everything is symmetric, we will focus throughout on the bid price b_n (the price at which the MM buys the asset, and the trader sells).

(a). Suppose we are analyzing an equilibrium in which the MMs' strategy is to **not** acquire information ($p_n = 0$ for $n = 1, 2$). Argue that in this case, $b_n = 1/2$ in equilibrium.

(b). Suppose MMn chooses never to become informed ($p_n = 0$), and MMm chooses always to become informed ($p_m = 1$). Suppose the uninformed MM sets prices $0 < b_n \leq a_n < 1$. Argue that in this case, the informed MM should optimally set bid (ask) price marginally above (below) the informed MM's bid (ask) price whenever $V = 1$ ($V = 0$). As for the ask (bid) price when $V = 1$ ($V = 0$), any price such that the informed MM will not sell (buy) the asset is optimal.

(c). Show that the lowest c such that no MM becomes informed in equilibrium is $c = 1/4$. That is to say, MMs will be uninformed for $c > 1/4$, but informed ($p_n > 0$ for at least one MM) for $c < 1/4$.

(d). Suppose MM1 becomes informed for sure ($p_1 = 1$) whereas MM2 does not become informed ($p_2 = 0$). Calculate the bid price set by MM2 in this case (calculate b_2) and argue that $p_1 = 1$ and $p_2 = 0$ can never be an equilibrium.

Hint: Consider three cases: $b_2 < 0$, $b_2 = 0$, and $b_2 > 0$ to find the optimal price b_2. Then calculate the gains to being informed versus being uninformed for both MMs.

(e). The previous question shows that it is not an equilibrium for one MM to be informed and the other not. Now, suppose both MMs become informed with the same interior probability:

$$p_1 = p_2 = p \in (0, 1).$$

Focus again on the bid side. When both MMs are potentially informed, the price-setting will be in mixed strategies. Suppose that if MMn is uninformed he uses the strategy

$$\sigma(b) = \mathbb{P}(b_n \leq b | \text{uninformed}),$$

and if MMn is informed and learns that the value is high ($V = 1$) he follows the strategy

$$\overline{\sigma}(b) = \mathbb{P}(b_n \leq b | \text{informed}, V = 1).$$

If MMn is informed and learns that the value is low ($V = 0$), he just sets $b_n = 0$ with probability one. Thus, the two MMs follow the same strategy. Notice that the strategies σ and $\overline{\sigma}$ indicate the probability that the MM sets a bid price below b.

Furthermore, suppose that there exist l and u with $0 < l < u < 1$ such that the following holds:

$$\sigma(0) = 0, \qquad \sigma(l) = 1, \qquad \overline{\sigma}(l) = 0, \qquad \overline{\sigma}(u) = 1.$$

Hence, the uninformed MM bids in the interval $[0, l]$, and the informed MM who learns that the value is high bids in the interval $[l, u]$.

Consider the gross profit (not counting potential information cost) from bidding b when a sell order arrives. Denote this $\Pi(b)$ for an uninformed MM and $\overline{\Pi}(b)$ for informed MM who observes $V = 1$. Show that if the MMs use the above strategy then:

$$\Pi(b) = \frac{1}{2}\left[p + (1-p)\sigma(b)\right](0 - b) + \frac{1}{2}[(1-p)\sigma(b)](1-b);$$
$$\overline{\Pi}(b) = [p\overline{\sigma}(b) + (1-p)](1-b).$$

Hint: For the uninformed MM, there are two possibilities: $V = 0$ and $V = 1$. Each possibility carries a different probability of winning. On the other hand, the informed MM who observed $V = 1$ knows the asset value.

(f). Argue informally that the uninformed MM must make zero profits: $\Pi(b) = 0$ for all $b \in [0, l]$. Then, together with $\sigma(l) = 1$, use this to find l.

(g). Assume that the informed MM will make a positive profit. Since he follows a mixed strategy, we must have $\overline{\Pi}(b) = \overline{\Pi} > 0$ for all $b \in [l, u]$. Use this together with $\overline{\sigma}(l) = 0$ and your answer to the previous question to find $\overline{\Pi}$.

(h). Since everything is symmetric in the model, the expected gross profit to an informed MM is the probability of a sell order times the gross profit conditional on a sell order, that is, it is $\frac{1}{2} \cdot \overline{\Pi}$. For $c \in (0, 1/4)$, we will have $p \in (0, 1)$. Since MMs use a mixed strategy in information acquisition, their net profits from acquiring information (gross profit less c) must be zero. Use this to find the equilibrium probability $p^*(c)$ of acquiring information as a function of information cost.

Problem 3

A great deal of the course has been devoted to analyzing how adverse selection drives pricing in financial markets. We have mainly assumed that only market makers faced adverse selection (for instance, the models by Glosten/Milgrom and Kyle), but we have also touched on the possibility that market makers can be informed. **Discuss, using real life examples, how adverse selection (on both sides of the market) may affect market outcomes.**

You should not limit yourself only to the arguments we have seen in class, but think more broadly about the question. You may wish, but are not obliged, to consider the following issues: (i) The distinction between market makers and traders is not always clear, e.g. in a limit order book you can choose between providing liquidity or taking it, you are not bound to a role a priori. (ii) Market makers are most often large institutions, traders may be large or small. (iii) Regulation on information sharing between different branches of large banks (so-called 'Chinese walls'). (iv) The role of high-frequency trading in market making/speculation and how this affects information-driven adverse selection.